THICKWALL CYLINDERS AND PRESS FITS

Chapter 3

Nomenclature

A	=	cross-sectional area, m^2
$\bar{a}$	=	coefficient of linear thermal expansion, $(^{\circ}C)^{-1}$
C_1, C_2	=	integration constants
c	=	radial clearance
d	=	diameter, m
E	=	modulus of elasticity, Pa
l	=	length, m
n_s	=	safety factor
P	=	force, N
p	=	pressure, Pa
r	=	radius, m
S_y	=	yield strength, Pa
T	=	torque, N-m
t_h	=	thickness, m
t_l	=	tolerance, m
Δt_m	=	temperature change, ºC
x,y	=	Cartesian co-ordinates, m
z	=	axial direction in cylindrical polar co-ordinates, m
α	=	cone angle, deg
β	=	body force per volume, N/m^3
Δ	=	diametral interference, m
δ	=	interference or displacement, m
ε	=	strain
θ	=	circumferential direction in cylindrical polar co-ordinates, rad
μ	=	coefficient of friction
ν	=	Poisson's ratio
ρ	=	density, kg/m^3
σ	=	normal stress, Pa
ω	=	angular velocity, rad/s

Chapter 3 – Thick-Walled Cylinders and Press Fits

3.1 Introduction

Cylindrical elements are often used as machinery elements. Often a cylindrical shaft in a round hole may serve a specific purpose. In designing these elements engineers are expected to specify class of fit or specific tolerances. The tolerances, clearances or interferances are specified according to the two tables on page 388 in the text.

Of primary concern in this course are the elements which are held together by the force caused by elastic deformation as one element is forced into another to form the part. The theory governing the design of these parts is developed from elastic theory applied to the stress – strain relationships in thick-walled cylinders. A wall is considered thick if it is $^1/_{10}$ the cylinder radius or more.

3.2 Thick-Walled Cylinders

3.2.1 Stress Distribution

In thick-walled cylinders under internal and/or external pressure, the principal stresses are σ_r and σ_t in the normal plane and σ_o in the longitudinal direction. The longitudinal σ_o is not significant because: (1) It is equal to zero in open-ended cylinders, (2) It falls between σ_r and σ_θ and, hence, does not enter commonly used maximum shear stress theory, (3) It is uniform across the cross-section and along the length so that it does not influence σ_r and σ_θ. Hence, the condition of plane stress exists.

The principal stress σ_r and σ_θ for a thick-walled cylinder under internal and/or external pressure as shown in Fig. 1.1 are given by the following equations derived in Chapter 10 of textbook:

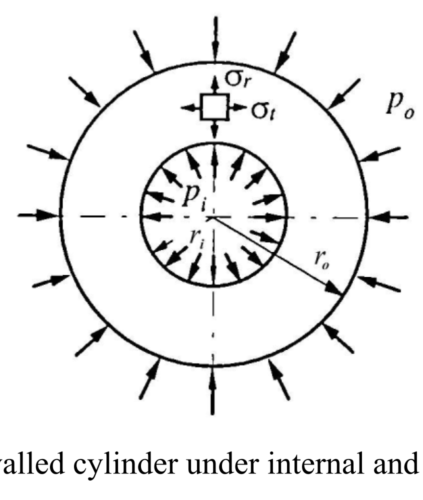
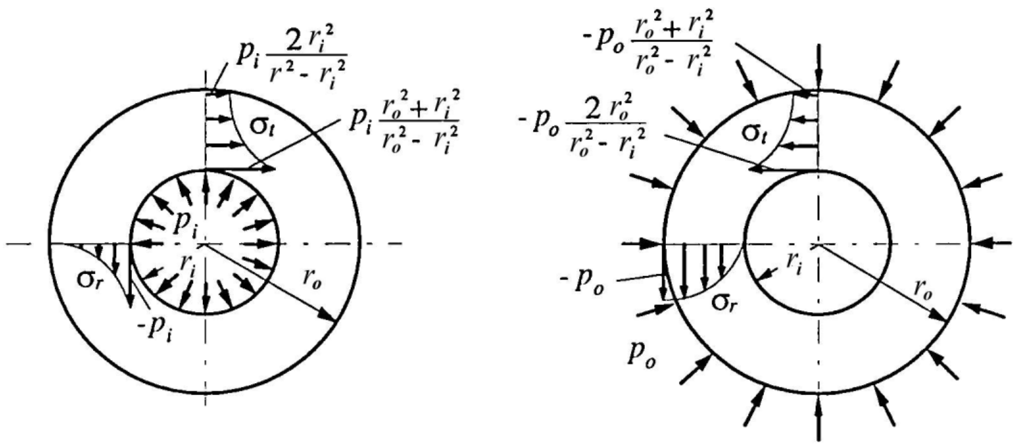


Fig. 3.1 Thick-walled cylinder under internal and external pressures

$$\sigma_r = \frac{r_i^2 p_i - r_o^2 p_o}{r_o^2 - r_i^2} - \frac{r_i^2 r_o^2 (p_i - p_o)}{r^2 (r_o^2 - r_i^2)} \tag{3.1}$$

$$\sigma_\theta = \frac{r_i^2 p_i - r_o^2 p_o}{r_o^2 - r_i^2} + \frac{r_i^2 r_o^2 (p_i - p_o)}{r^2 (r_o^2 - r_i^2)} \tag{3.2}$$

The stress distribution is plotted in Fig. 3.2 for p_i and p_o acting alone.



(a) p_i acting alone (b) p_o acting alone

Fig. 3.2 Stress distribution

The radial displacement δ_r is given by

$$\delta_r = \frac{r_i}{E}(\sigma_\theta - \nu\sigma_r) \tag{3.3}$$

where ν is Poisson's ratio and E is the modulus of elasticity of the material.

3.2.2 Limiting Internal Pressure for Solid Cylinders

The allowable internal pressure for solid cylinders is limited not by wall thickness but by the yield strength of the cylinder material. To illustrate this fact, assume ductile material and the maximum shear stress theory for yielding, outside pressure $p_o = 0$.

Yielding will occur at the inner surface of the cylinder ($r = r_i$) when

$$\sigma_r - \sigma_\theta = 2\tau_y$$

Substitute for σ_r and σ_θ from Eqs. (3.1) and (3.2) with $r = r_i$, $p_o = 0$ results

$$2\tau_y = 2\frac{r_o^{\,2} p_i}{r_o^{\,2} - r_i^{\,2}}$$

or

$$r_o = r_i\sqrt{\frac{\tau_y}{\tau_y - p_i}} \qquad \textbf{(3.4)}$$

It is seen as p_i approaches τ_y, the material yield strength, the required radius b becomes infinite. Hence, the allowable internal pressure can not be made arbitrarily high by making the cylinder wall arbitrarily thick. Furthermore, it is obvious that yielding will commence when the bulk of the material, with increasing r, is at low stress. Hence, solid thick walled cylinders use material inefficiently. Material usage can be greatly improved by properly designed compound thick cylinders.

3.3 Compound Cylinders

A compound cylinder is made by press-fitting one or more jackets around an inner cylinder. This causes a residual compressive stress σ_θ at the cylinder bore (at $r = r_i$) when the internal pressure is zero. Then when the working pressure p_i is applied, the resulting tangential stress is superimposed with the residual tangential stress leaving a reduced net σ_θ.

3.3.1 Shrink and Press Fit Stresses

In a shrink or press fit, the residual stresses generated depend on the amount of interference. Assume two cylinders as shown in Fig. 3.3, in which

$$D_o - D_i = 2\delta_r = \text{diametral interference}$$

In order to be assembled at radius r_f, the larger tube must displace outward at c by the amount u_{ro}, the smaller tube must displace inward at r_f by the amount δ_{ri}. The total displacement must equal the diametral interference, Δ, or

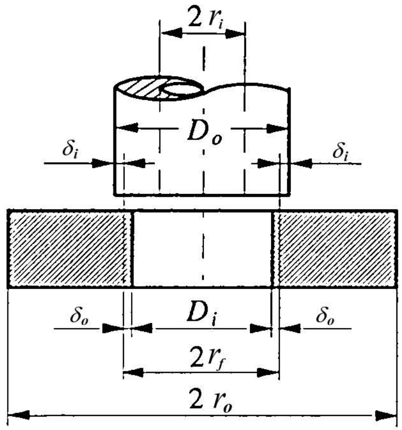


Fig. 3.3 Interference of metal in press fit

$$\Delta = 2(\delta_{ro} - \delta_{ri}) \tag{3.5}$$

The displacement is given by Eq. (3.3), that is, for outer tube (or disk)

$$\delta_{ro} = \frac{r_f}{E}(\sigma_\theta - \nu\sigma_\theta)$$

Substituting Eqs. (3.1) and (3.2) for $p_o = 0$, $r_i = r_f$, $r = r_f$, $p_i = p_c$, results in

$$\delta_{ro} = \frac{r_f}{E} p_c \left(\frac{r_o^2 + r_f^2}{r_o^2 - r_f^2} + \nu \right) \tag{3.6}$$

Similarly, for inner tube (or shaft)

$$\delta_{ri} = \frac{r_f}{E}(\sigma_\theta - \nu\sigma_\theta)$$

Substituting Eqs. (3.1) and (3.2) for $p_i = 0$, $r_o = r_f$, $r = r_f$, $p_o = p_c$, results in

$$\delta_{ri} = -\frac{r_f}{E} p_c \left(\frac{r_f^2 + r_i^2}{r_f^2 - r_i^2} - \nu \right) \tag{3.7}$$

If the two cylinders are of the same material, substituting Eqs. (3.6) and (3.7) into Eq. (3.5) gives,

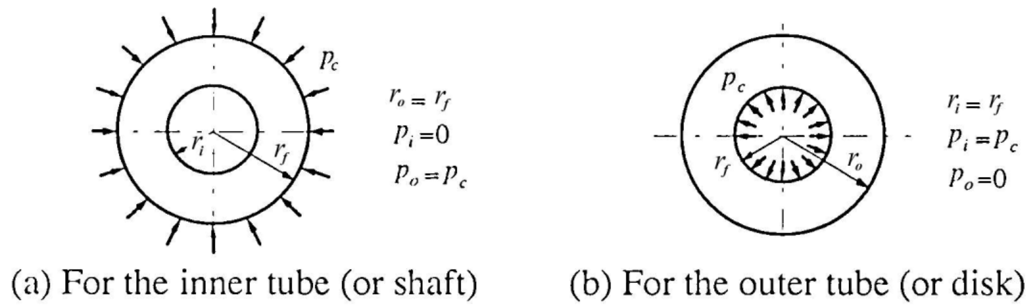
$$\Delta = \frac{4r_f^3}{E} p_c \left[\frac{r_o^2 - r_i^2}{(r_f^2 - r_i^2)(r_o^2 - r_f^2)} \right]$$

(3.8)

Thus the contact pressure, p_c, due to interference alone, is

$$p_c = \frac{E\Delta\left(r_f^{\,2} - r_i^{\,2}\right)\left(r_o^{\,2} - r_f^{\,2}\right)}{4 r_f^{\,3}\left(r_o^{\,2} - r_i^{\,2}\right)} \qquad \textbf{(3.9)}$$

The interference stresses can now be calculated by Eqs. (3.1) and (3.2) with the following substitutions:



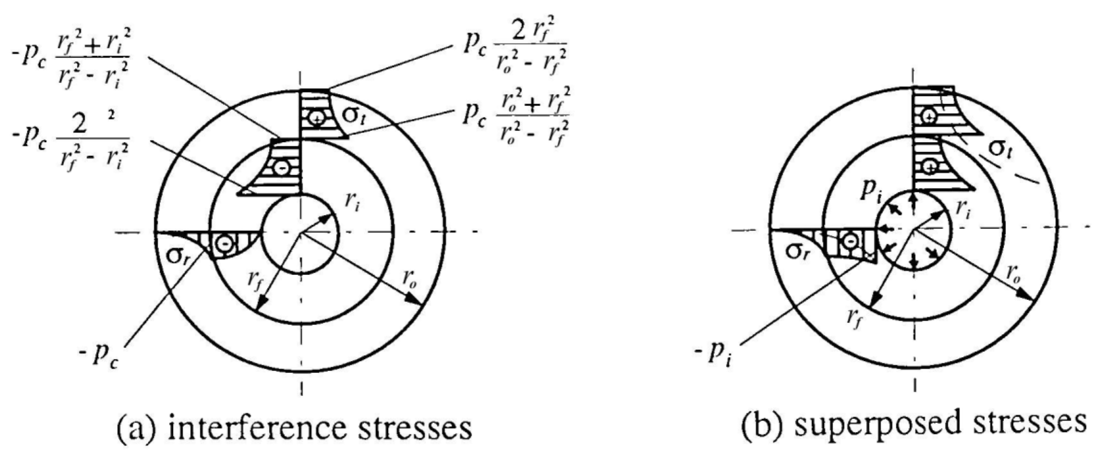


Fig. 3.4 Calculation of the interference stresses

Fig. 3.5 Stress distribution

The calculated interference stresses are shown in Fig. 3.5(a). Stresses due to p_i can be easily superposed as shown in Fig. 3.5(b).

Example 1: A 200-mm-diameter steel shaft is to have a press fit in a 500-mm-diameter cast iron disk. The maximum tangential stress in the disk is to be 35 MPa. The modulus of elasticity for steel is 206,900 MPa and half this amount for cast iron. Poisson's ratio is equal to 0.3.

Find:

(a) the required diametral interference of metal.

(b) the force required to press the parts together if the coefficient of friction is equal to 0.12 and the disk is 250 mm thick in the axial direction.

(c) the torque which the joint could carry because of the shrink fit pressure.

Solution: (a) By Eq. (3.2) or from Fig. 3.2(a): with r_i = 100 mm, r_o = 250 mm, and p_o = 0, r = r_i, σ_θ = 35 MPa

$$p_c = p_i = \frac{r_o^2 - r_i^2}{r_o^2 + r_i^2}\sigma_t = \frac{250^2 - 100^2}{250^2 + 100^2} \times 35 = 25.3\ \text{MPa}$$

For disk: By Eq. (3.6): with r_f = 100 mm, r_o = 250 mm

$$\delta_{ro} = \frac{r_f}{E} p_c \left(\frac{r_o^2 + r_f^2}{r_o^2 - r_f^2} + \nu \right) = \frac{100 \times 25.3}{\frac{2.069 \times 10^5}{2}} \left(\frac{250^2 + 100^2}{250^2 - 100^2} + 0.3 \right) = 0.0412\ \text{mm}$$

outward, that is, increase in hole radius

For shaft: By Eq. (3.7): with r_f = 100 mm, r_i = 0

$$\delta_{ri} = -\frac{r_f}{E} p_c \left(\frac{r_f^2 + r_i^2}{r_f^2 - r_i^2} - \nu \right) = -\frac{100 \times 25.3}{2.069 \times 10^5}(1 - 0.3) = -0.0086\ \text{mm}$$

Inward, that is, decrease in shaft radius

By Eq. (3.5): $\Delta = 2\ (\delta_{ro} - \upsilon_{ri}) = 2\ (0.0412 + 0.0086) = 0.100$ mm

(b): Force required for assembly: $F = 200\ \pi\ 250 \times 25.3 \times 0.12 = 477{,}000$ N

(c): Torque carried by press fit: $T = 477{,}000 \times 100 = 47{,}700{,}000$ Nmm

3.3.2 Design of Compound Cylinders

Given: internal pressure p_i, bore radius r_i, and outer radius r_o.
Required: optimum radius r_f and optimum pressure p_c.

Assume the maximum shear stress theory as a basis of the design.

$$\tau_m = \frac{\sigma_t - \sigma_r}{2}$$

Assume that for optimum design, the maximum shear stress in the inner tube, τ_{mi}, is equal to the maximum shear stress in the outer tube, τ_{mo}.

$$\tau_{mi} = \tau_{mo}$$

Evaluate τ_{mi} at $r = r_i$, superposing stresses due to p_c and p_i.
Evaluate τ_{mo} at $r = r_f$, superposing stresses due to p_c.
Equate τ_{mi} and τ_{mo} and obtain

$$p_c = p_i \frac{\dfrac{r_o^2 (r_f^2 - r_i^2)}{r_f^2 (r_o^2 - r_i^2)}}{\left(\dfrac{r_f^2}{r_f^2 - r_i^2} + \dfrac{r_o^2}{r_o^2 - r_f^2}\right)} \tag{3.10}$$

Substitute p_c in τ_m and obtain

$$\tau_m = \frac{p_i r_o^2 r_f^2}{r_f^2\left(r_o^2 - r_f^2\right) + r_o^2\left(r_f^2 - r_i^2\right)} \tag{3.11}$$

Take $\dfrac{d\tau_m}{d r_f} = 0$ and obtain optimum c.

$$r_f = \sqrt{r_i r_o} \tag{3.12}$$

Substitute Eq. (3.12) into Eq. (3.10) and obtain optimum p_c.

$$p_c = \frac{p_i\left(r_o - r_i\right)}{2\left(r_o + r_i\right)} \tag{3.13}$$

Substitute p_c from Eq. (3.13) into Eq. (3.11) to obtain a convenient expression for τ_{max} in optimum design.

$$\tau_m = \frac{p_i r_o}{2(r_o - r_i)} \tag{3.14}$$